

# *Predictive Analytics Workshop*

## **Data Cleaning / Exploration and Feature Engineering**

John Murdzek, FSA

Senior Experience Studies Actuary

Genworth Financial

Joe Long

Assistant Actuary and Data Scientist

Milliman, Minneapolis

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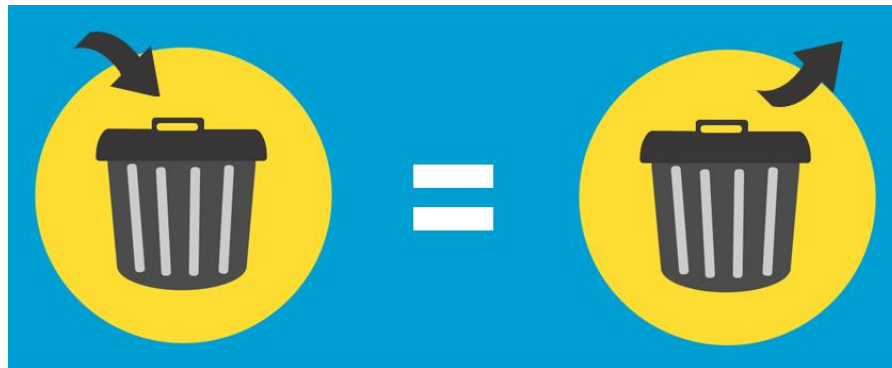
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- Now we have data let's start modeling right?  
Nope....
- Garbage in is garbage out, make sure your data is in good shape
- ~80% of your time will be spent here

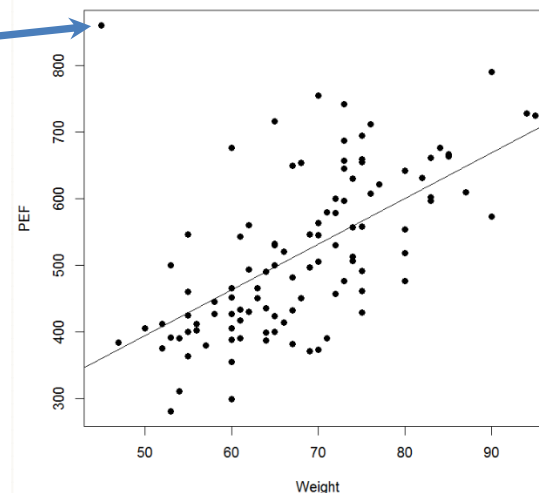


# Getting started



- It's important to make sure your data are clean and ready to go for your modeling project.
- Have you viewed your driver and response variables?
- Does your data have any outliers, or blanks?

[Run R 0100: 1.0.1 – 1.0.7]

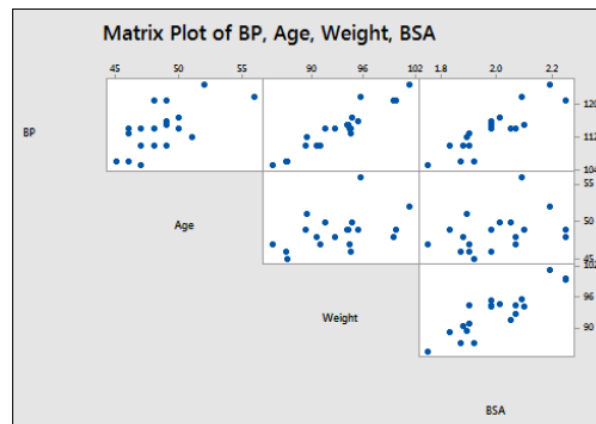


Source:  
<http://stats.stackexchange.com/questions/194783/extreme-values-in-the-data>

# Data relationships



- If you notice patterns in your data that are strange find out why and try and fix them.
- Have you viewed relationships amongst your driver and response variables?
- Are some driver variables so related that they do not appear to add any value?



Source:  
<https://onlinecourses.science.psu.edu/stat501/node/347>

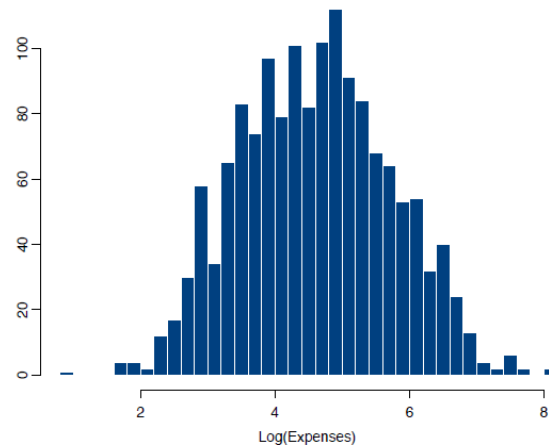
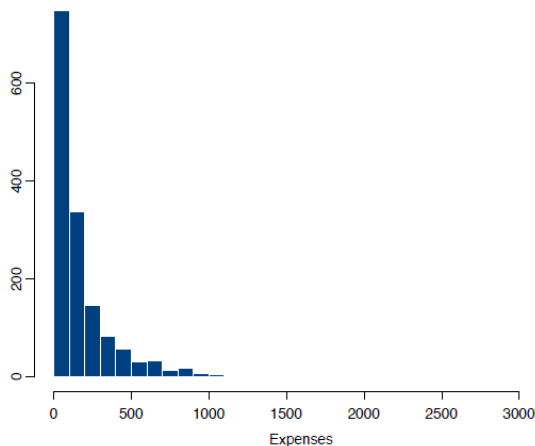


- Sometimes your data may not be good enough quality to use in a modeling project.
  - Are the data granular enough for your modeling purpose?
  - Are the data credible enough? Can industry data help out?
  - Do you find too many outliers, blanks, and / or other suspicious patterns?
  - Do you find that certain relationships you expect to be true, are violated by simple views of your data?
- [Run R 0100: 1.0.8 – 1.0.13]

# Variable transformations



- Data is clean – Can we make any additional variables?
- Start simple – We can always circle back.
- Would any transformations of your variables better serve you?



Source:  
[http://www.kenbenoit.  
net/courses/ME104/lo  
gmodels2.pdf](http://www.kenbenoit.net/courses/ME104/lo<br/>gmodels2.pdf)





- The most populous part of your data is what can be considered the “base level” characteristic.
- For example, if there are more claim terminations for females than males, then females would be the desirable base level characteristic of the gender driver variable.
- This concept will become more important in the next section.

*“The base level should not be sparse. ... Any level which is not sparse is an appropriate base level.”*

*– Piet de Jong and Gillian Z. Heller, “Generalized Linear Models for Insurance Data”*



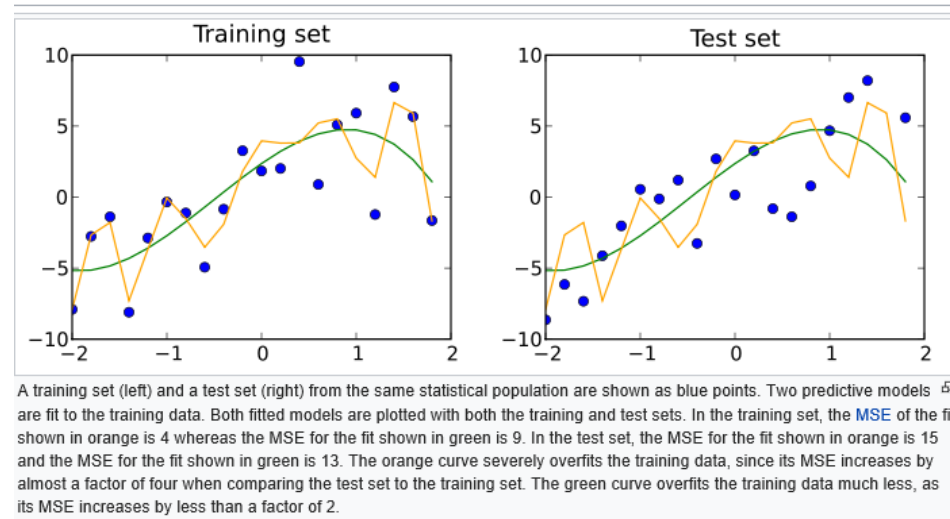
- Key data fields
  - GroupIndicator
  - Cov\_Type\_Bucket
  - ClaimDuration
  - Gender
  - ClaimType
  - Exposure
  - Terminations
  - Incurred\_Year
  - IncurredAgeBucket
  - Region
  - Diagnosis\_Category
  - TQ\_Status
  - Infl\_Rider\_Bucket
  - Max\_Ben\_Bucket

# Splitting the datasets



- Split data into training and validation data.
- What proportion of your data can you afford to segment out of your training data analysis?
- Can you afford to also make a test dataset?

[Run R 0150]



Source:  
[https://en.wikipedia.org/wiki/Test\\_set](https://en.wikipedia.org/wiki/Test_set)

# *Predictive Analytics Workshop*

## **GLM / Poisson Modeling**

John Murdzek, FSA  
Senior Experience Studies Actuary  
Genworth Financial

March 21, 2018



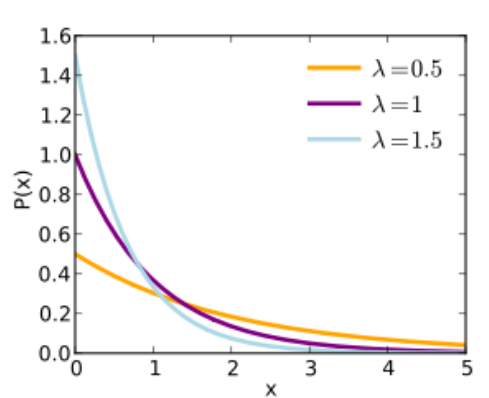
**18th Annual Intercompany Long Term Care Insurance Conference**



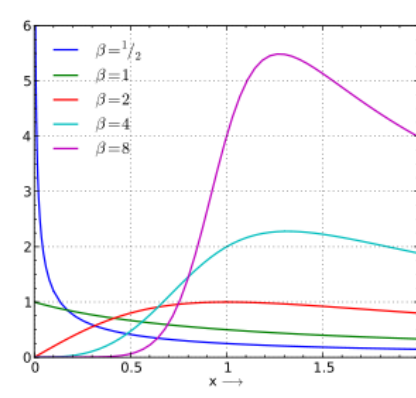
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- Start simple to get a feel for your data.
- Do you have some *a priori* notions as to what relationships should exist in your dataset?
- Do you have a sense as to how the hazard rate function might look?
- R can help you fit models based on your answers to the above.



Exponential failure density functions. Each of these has a (different) constant hazard function (see text).



Hazard function  $h(t)$  plotted for a selection of log-logistic distributions.

Source:  
[https://en.wikipedia.org/wiki/Failure\\_rate](https://en.wikipedia.org/wiki/Failure_rate)



- Initial test GLMs can be run to determine which variables are contributing most to explaining the variation of the CTRs.
- But first the data need to be prepared to make the Poisson regression possible.
- The data preparation steps condense what was done in the pre-work and create “factor” variables which are important to the GLM code.

[Run R 0240: 2.4.1 – 2.4.10]

# Simple Poisson GLM results



- Fit gender, situs, and claim duration
- The residuals can help indicate over-dispersion.
- The “Estimates” are the GLM coefficients. Their exponentiated values produce the CTR of any effect that is desired.
- The standard errors and p-values tell you whether the estimate is significant.

[Run R 0240: 2.4.11 – 2.4.12]

```
+ cnisq_stat = STATISTIC ** 2)
> summary(results_ps)

Call:
glm(formula = SumTerminations ~ relevel(Gender.f, "Female") +
    relevel(claimType.f, "HHC") + relevel(cldurBucket.f, "Mos1-3"),
    family = poisson, data = ctr_data_ps, offset = LnExposure)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.2132  -0.6157  -0.4565  -0.1808   4.1074

Coefficients:
                Estimate Std. Error z value Pr(>|z|)
(Intercept)      -2.48972    0.03009  -82.745 < 2e-16
relevel(Gender.f, "Female")Male    0.29472    0.01845   15.970 < 2e-16
relevel(claimType.f, "HHC")ALF     -0.34411    0.02386  -14.422 < 2e-16
relevel(claimType.f, "HHC")NH      0.11367    0.02038    5.578 2.44e-08
relevel(cldurBucket.f, "Mos1-3")Mo4-12 -1.03715    0.03157  -32.851 < 2e-16
relevel(cldurBucket.f, "Mos1-3")Yrs2-6 -1.40913    0.03048  -46.226 < 2e-16
relevel(cldurBucket.f, "Mos1-3")Yrs7+ -1.44508    0.05953  -24.274 < 2e-16
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 59074  on 69418  degrees of freedom
Residual deviance: 56591  on 69412  degrees of freedom
AIC: 80812

Number of Fisher Scoring iterations: 6

> |
```





- Include some new variables in the GLM from the list below, in addition to Gender, ClaimType, and ClmDurBucket:
  - IncurredAgeBucket.f
  - Region.f
  - Diagnosis\_Category.f
  - TQ\_Status.f
  - Infl\_Rider\_Bucket.f
  - Max\_Ben\_Bucket.f
- Spend 15 - 20 minutes determining which variables you might want to incorporate

# Poisson GLM variable selection



- Age is important and significant.
- Region appears not to be as helpful.
- The diagnosis categories could probably use some grouping.
- Tax qualification appears not to add too much value.
- The inflation options appear not to be too different from one another.
- Benefit period is important and could use some grouping.

[Run R 0240: 2.4.14]

```
call:
glm(formula = SumTerminations ~ relevel(Gender.f, "Female") +
  relevel(ClaimType.f, "HHC") + relevel(IncurredAgeBucket.f,
    "80 to 84") + relevel(ClmdurBucket.f, "Mos1-3") + relevel(Region.f,
    "Unknown") + relevel(Diagnosis_Category.f, "Unknown") + relevel(TQ_Status.f,
    "U") + relevel(Infl_Rider_Bucket.f, "Unknown") + relevel(Max_Ben_Bucket.f,
    "Unknown"), family = poisson, data = ctr_data_ps, offset = LnExposure)
```

```
Deviance Residuals:
    Min       1Q   Median       3Q      Max
-3.3828  -0.6052  -0.4258  -0.1796   4.0314
```

Coefficients:

(Intercept)	-3.0637613	0.5048068	-6.069	1.29e-09
relevel(Gender.f, "Female")Male	0.3168230	0.0187292	16.916	< 2e-16
relevel(ClaimType.f, "HHC")ALF	-0.2261186	0.0259880	-8.701	< 2e-16
relevel(ClaimType.f, "HHC")NH	0.2058818	0.0211961	9.713	< 2e-16
relevel(IncurredAgeBucket.f, "80 to 84")60 to 64	0.3104159	0.0497131	6.244	4.26e-10
relevel(IncurredAgeBucket.f, "80 to 84")65 to 69	0.1359209	0.0385241	3.528	0.000418
relevel(IncurredAgeBucket.f, "80 to 84")70 to 74	0.0840069	0.0301986	2.782	0.005406
relevel(IncurredAgeBucket.f, "80 to 84")75 to 79	0.0431267	0.0258587	1.668	0.095360
relevel(IncurredAgeBucket.f, "80 to 84")85 to 89	0.0416842	0.0286104	1.457	0.145128
relevel(IncurredAgeBucket.f, "80 to 84")90+	0.0522087	0.0447138	1.168	0.242960
relevel(IncurredAgeBucket.f, "80 to 84")LT 60	0.3130447	0.0546117	5.732	9.91e-09
relevel(ClmdurBucket.f, "Mos1-3")Mo4-12	-0.8780505	0.0319685	-27.466	< 2e-16
relevel(ClmdurBucket.f, "Mos1-3")Yrs2-6	-1.1326129	0.0313270	-36.154	< 2e-16
relevel(ClmdurBucket.f, "Mos1-3")Yrs7+	-1.1449662	0.0606209	-18.887	< 2e-16
relevel(Region.f, "Unknown")Mid-West	0.4501381	0.0636294	7.074	1.50e-12
relevel(Region.f, "Unknown")Northeast	0.4850839	0.0650068	7.462	8.52e-14
relevel(Region.f, "Unknown")South	0.5135551	0.0623401	8.238	< 2e-16
relevel(Region.f, "Unknown")West	0.4365562	0.0637572	6.847	7.53e-12
relevel(Diagnosis_Category.f, "Unknown")Alzheimer's	-0.4626133	0.0549960	-8.412	< 2e-16
relevel(Diagnosis_Category.f, "Unknown")Arthritis	-0.3469784	0.0623161	-5.568	2.58e-08
relevel(Diagnosis_Category.f, "Unknown")Cancer	1.0092363	0.0572849	17.618	< 2e-16
relevel(Diagnosis_Category.f, "Unknown")Circulatory	-0.0382711	0.0620208	-0.617	0.537189
relevel(Diagnosis_Category.f, "Unknown")Congenital	0.5163128	0.2107075	2.450	0.014271
relevel(Diagnosis_Category.f, "Unknown")Diabetes	-0.3727338	0.1043621	-3.572	0.000355
relevel(Diagnosis_Category.f, "Unknown")Digestive System	0.1708742	0.0989065	1.728	0.084054
relevel(Diagnosis_Category.f, "Unknown")Endocrine/Immunity System	-0.1164194	0.1361622	-0.855	0.392548
relevel(Diagnosis_Category.f, "Unknown")Genitourinary System	0.0009937	0.1030113	0.010	0.992303
relevel(Diagnosis_Category.f, "Unknown")Hypertension	-0.4392938	0.1362949	-3.223	0.001268
relevel(Diagnosis_Category.f, "Unknown")Ill-Defined And Miscellaneous Conditions	-0.3472433	0.0718307	-4.834	1.34e-06
relevel(Diagnosis_Category.f, "Unknown")Injury	0.0532918	0.0608328	0.876	0.381010
relevel(Diagnosis_Category.f, "Unknown")Mental	-0.6377687	0.1445785	-4.411	1.03e-05
relevel(Diagnosis_Category.f, "Unknown")Nervous System And Sense Organs	-0.4544683	0.0626192	-7.258	3.94e-13
relevel(Diagnosis_Category.f, "Unknown")Pregnancy Disorders	0.1637342	0.3811007	0.430	0.667461
relevel(Diagnosis_Category.f, "Unknown")Respiratory	0.1404179	0.0693015	2.026	0.042746
relevel(Diagnosis_Category.f, "Unknown")Skin And Subcutaneous Tissue	-0.2243730	0.1703392	-1.317	0.187767
relevel(Diagnosis_Category.f, "Unknown")Stroke	-0.4473993	0.0599936	-7.457	8.82e-14
relevel(TQ_Status.f, "U")N	-0.3063513	0.0498924	-6.140	3.94e-10
relevel(TQ_Status.f, "U")Q	-0.3504302	0.0239963	-14.604	< 2e-16
relevel(Infl_Rider_Bucket.f, "Unknown")GPO	0.4926762	0.5004115	0.985	0.324849
relevel(Infl_Rider_Bucket.f, "Unknown")Inflation	0.4844602	0.5003174	0.968	0.332892
relevel(Infl_Rider_Bucket.f, "Unknown")None	0.5038136	0.5004384	1.007	0.314058
relevel(Max_Ben_Bucket.f, "Unknown")1 to 2	-0.1304479	0.0552113	-2.363	0.018142
relevel(Max_Ben_Bucket.f, "Unknown")3 to 4	-0.2401490	0.0525437	-4.570	4.87e-06
relevel(Max_Ben_Bucket.f, "Unknown")5 +	-0.3205315	0.0579384	-5.532	3.16e-08
relevel(Max_Ben_Bucket.f, "Unknown")LT 1	0.1875213	0.0998139	1.879	0.060284
relevel(Max_Ben_Bucket.f, "Unknown")Unlimited	-0.4693533	0.0535686	-8.762	< 2e-16



- A Type III analysis can also be helpful in understanding if an entire driver variable is important and significant.

[Run R 0240: 2.4.15]

- The table shows the impact to Deviance and AIC for removing a driver variable.

```
> drop1(results_ps,~,test="chisq")
Single term deletions

Model:
SumTerminations ~ relevel(Gender.f, "Female") + relevel(ClaimType.f,
  "HHC") + relevel(IncurredAgeBucket.f, "80 to 84") + relevel(ClmdurBucket.f,
  "Mos1-3") + relevel(Region.f, "Unknown") + relevel(Diagnosis_Category.f,
  "Unknown") + relevel(TQ_Status.f, "U") + relevel(Infl_Rider_Bucket.f,
  "Unknown") + relevel(Max_Ben_Bucket.f, "Unknown")

```

	Df	Deviance	AIC	LRT	Pr(>Chi)
<none>		53708	78008		
relevel(Gender.f, "Female")	1	53987	78285	279.11	< 2.2e-16 ***
relevel(ClaimType.f, "HHC")	2	53976	78272	268.05	< 2.2e-16 ***
relevel(IncurredAgeBucket.f, "80 to 84")	7	53770	78056	61.79	6.627e-11 ***
relevel(ClmdurBucket.f, "Mos1-3")	3	54790	79084	1081.68	< 2.2e-16 ***
relevel(Region.f, "Unknown")	4	53775	78067	67.01	9.696e-14 ***
relevel(Diagnosis_Category.f, "Unknown")	18	56001	80264	2292.62	< 2.2e-16 ***
relevel(TQ_Status.f, "U")	2	53918	78213	209.51	< 2.2e-16 ***
relevel(Infl_Rider_Bucket.f, "Unknown")	3	53710	78004	1.83	0.608
relevel(Max_Ben_Bucket.f, "Unknown")	5	53916	78206	207.89	< 2.2e-16 ***

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> |
```

- All driver variables are statistically significant, except for the inflation rider variable.



- Variable selection decisions are potentially subjective ones. For this analysis, the following decisions are made:
  - The inflation rider driver does not contribute much explanatory power, and can therefore be eliminated.
  - Diagnosis and benefit period should probably be condensed to a more manageable set of values, and should be kept in the model.
  - The tax qualified status and region drivers are eliminated because the coefficients are so similar.

# Poisson GLM smoothing



- The calendar year variable's pattern is somewhat erratic, so it might make sense to smooth it out.
- A logarithmic variable is used in its place.
- This also gives the model more parsimony (lower AIC), and the potential to extrapolate by calendar year.

```
> results_ps <- glm(SumTerminations ~
+   relevel(Gender.f, "Female")
+   + relevel(ClaimType.f, "HHC")
+   + relevel(Diagnosis2.f, "Non-Mental")
+   + relevel(BP2.f, "Non-Life")
+   + relevel(IncurredAgeBucket.f, "80 to 84")
+   + relevel(ClmdurBucket.f, "Mos1-3")
+   + LnIncyr,
+   data = ctr_data_ps,
+   family = poisson,
+   offset = LnExposure
+ )
> summary(results_ps)
```

Call:  
glm(formula = SumTerminations ~ relevel(Gender.f, "Female") +  
 relevel(ClaimType.f, "HHC") + relevel(Diagnosis2.f, "Non-Mental") +  
 relevel(BP2.f, "Non-Life") + relevel(IncurredAgeBucket.f,  
 "80 to 84") + relevel(ClmdurBucket.f, "Mos1-3") + LnIncyr,  
 family = poisson, data = ctr\_data\_ps, offset = LnExposure)

Deviance Residuals:

	Min	1Q	Median	3Q	Max
	-3.5788	-0.9097	-0.4351	0.4983	3.7309

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	-2.25398	0.11854	-19.015	< 2e-16
relevel(Gender.f, "Female")Male	0.28387	0.01850	15.345	< 2e-16
relevel(ClaimType.f, "HHC")ALF	-0.26751	0.02429	-11.013	< 2e-16
relevel(ClaimType.f, "HHC")NH	0.14036	0.02064	6.799	1.05e-11
relevel(Diagnosis2.f, "Non-Mental")Mental	-0.39533	0.02117	-18.674	< 2e-16
relevel(BP2.f, "Non-Life")Lifetime	-0.20134	0.02127	-9.467	< 2e-16
relevel(IncurredAgeBucket.f, "80 to 84")60 to 64	0.34336	0.04849	7.081	1.43e-12
relevel(IncurredAgeBucket.f, "80 to 84")65 to 69	0.17628	0.03752	4.698	2.63e-06
relevel(IncurredAgeBucket.f, "80 to 84")70 to 74	0.09227	0.02966	3.111	0.00186
relevel(IncurredAgeBucket.f, "80 to 84")75 to 79	0.03985	0.02572	1.550	0.12124
relevel(IncurredAgeBucket.f, "80 to 84")85 to 89	0.06284	0.02847	2.207	0.02732
relevel(IncurredAgeBucket.f, "80 to 84")90+	0.07111	0.04426	1.607	0.10816
relevel(IncurredAgeBucket.f, "80 to 84")LT 60	0.38168	0.05366	7.113	1.14e-12
relevel(ClmdurBucket.f, "Mos1-3")Mos4-12	-1.00810	0.03163	-31.872	< 2e-16
relevel(ClmdurBucket.f, "Mos1-3")Yrs2-6	-1.38078	0.03096	-44.597	< 2e-16
relevel(ClmdurBucket.f, "Mos1-3")Yrs7+	-1.46500	0.06334	-23.130	< 2e-16
LnIncyr	-0.07028	0.04154	-1.692	0.09067

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

Null deviance: 10567.7 on 6221 degrees of freedom  
Residual deviance: 7525.7 on 6205 degrees of freedom  
AIC: 16995

Number of Fisher Scoring iterations: 6

```
>
```

[Run R 0240: 2.4.16 – 2.4.20]

# Poisson GLM cross-terms



- The introduction of cross-terms can also help improve the model fit.
- Cross-terms are introduced between:
  - Calendar year / Age
  - Calendar year / Duration
- The model fit is improved, and the extra parameter “expense” appears to be justified (lower AIC).

```
Console ~/R Projects/Examples/Output/ ↗
+ relevel(Diagnosis2.f, "Non-Mental")
+ relevel(BP2.f, "Non-Life")
+ LnInCyr * relevel(InCurredAgeBucket.f, "80 to 84")
+ LnInCyr * relevel(ClmburBucket.f, "Mos1-3"),
data = ctr_data_ps,
family = poisson,
offset = LnExposure
> summary(results_ps)

Call:
glm(formula = SumTerminations ~ relevel(Gender.f, "Female") +
  relevel(ClaimType.f, "HHC") + relevel(Diagnosis2.f, "Non-Mental") +
  relevel(BP2.f, "Non-Life") + LnInCyr * relevel(InCurredAgeBucket.f,
    "80 to 84") + LnInCyr * relevel(ClmburBucket.f, "Mos1-3"),
  family = poisson, data = ctr_data_ps, offset = LnExposure)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-3.4506   -0.9082   -0.4417    0.5102    3.7714

Coefficients:
(Intercept)                                -0.07489
relevel(Gender.f, "Female"):Male            0.28770
relevel(ClaimType.f, "HHC"):ALF             -0.26359
relevel(ClaimType.f, "HHC"):NH              0.14420
relevel(Diagnosis2.f, "Non-Mental")          -0.39268
relevel(BP2.f, "Non-Life"):Lifetime         -0.20009
LnInCyr                                     -0.88478
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):60 to 64 -2.25650
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):65 to 69 -2.52326
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):70 to 74 -1.11712
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):75 to 79 -0.21513
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):85 to 89  0.43962
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):90+      0.73858
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):LT 60    -3.06430
relevel(ClmburBucket.f, "Mos1-3"):Mo4-12      -2.70735
relevel(ClmburBucket.f, "Mos1-3"):Yrs2-6      -3.07292
relevel(ClmburBucket.f, "Mos1-3"):Yrs7+      -2.84736
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):60 to 64  0.98602
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):65 to 69  1.03356
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):70 to 74  0.46082
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):75 to 79  0.09385
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):85 to 89 -0.13673
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):90+      -0.23860
LnInCyr:relevel(InCurredAgeBucket.f, "80 to 84"):LT 60    1.30804
LnInCyr:relevel(ClmburBucket.f, "Mos1-3"):Mo4-12          0.62960
LnInCyr:relevel(ClmburBucket.f, "Mos1-3"):Yrs2-6          0.62937
LnInCyr:relevel(ClmburBucket.f, "Mos1-3"):Yrs7+           0.53373
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

Null deviance: 10567.7 on 6221 degrees of freedom
Residual deviance: 7394.7 on 6195 degrees of freedom
AIC: 16884

Number of Fisher Scoring iterations: 6
> |
```

[Run R 0240: 2.4.21]



- A Type I analysis can be run, which describes the variation explained by each driver variable in a sequential order. A different ordering might give you a different result.
- A Type III analysis can also be run, which describes reduction in the variation explained by removing one driver variable at a time. This is order-independent.

[Run R 0240: 2.4.22 – 2.4.25]





- If we use too many parameters in the GLM, we will over-fit the model.
- The data will be fit “too well” by the model, and will probably not test well in a separate random sample.

**[Run R 0240: 2.4.26 – 2.4.27]**





- We can replace the exposures ( $t$ ) in the GLM with an expected basis, like an industry table (or benchmark).
- The GLM coefficients are now in reference to the benchmark.
- Courtesy of Milliman:

Standard model:  $\ln(\mu) = \ln(t) + \beta_0 + \sum \beta_i x_i$

Update model:  $\ln(\mu) = \ln(t * benchmark) + \sum \beta_i x_i$

$$\hat{\mu}_j = t * benchmark * e^{\hat{\beta}_1 x_1} \times \dots \times e^{\hat{\beta}_k x_k}$$

[Run R 0250: 2.5.1 – 2.5.15]